

# Fondamenti della Meccanica Quantistica

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lezione 2

## Lezione 2

I problemi della misura, della località e dell'ontologie  
in Meccanica Quantistica

Cap. 3, 4, 5 di "Foundations of Quantum Mechanics" di T. Norsen.

Inizio lezione: riassunto delle puntate precedenti

### IL PROBLEMA DELLA MISURA

• La descrizione quantistica della misura

### POSTULATI DELLA MECCANICA QUANTISTICA

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x,t) \Psi(x,t) \quad \text{postulato}$$

postulato: regole di Born

$$\rho(x) = |\Psi(x,t)|^2 \quad \text{densità di probabilità}$$

(altri: operatori-osservabili; stati quantici) • Definizione completa dei postulati allegata e fine documento.

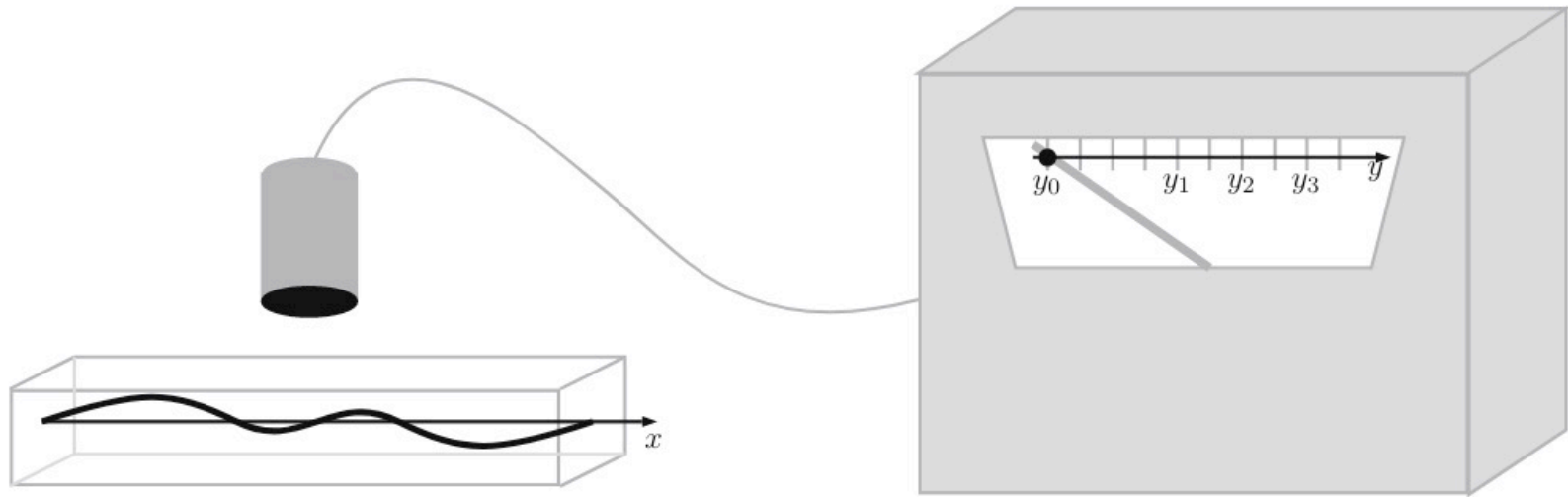
### POSTULATO DEL COLLASSO DELLA FUNZIONE D'ONDA

$$\Psi = \sum_i c_i \psi_i, \quad \text{dove } \hat{A} \psi_i = A_i \psi_i \\ \text{e } P(A_i) = |c_i|^2$$

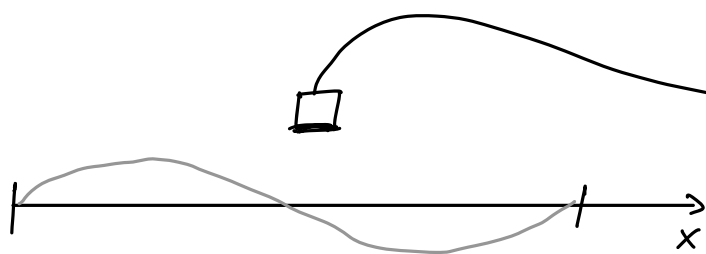
Se una misura comincia al tempo  $t_1$  e finisce al tempo  $t_2$   
potremmo scrivere

$$\Psi(t_1) = \sum_i c_i \psi_i \rightarrow \Psi(t_2) = \psi_m$$

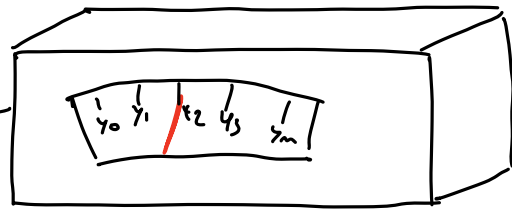
- 1) c'è un'evoluzione temporale molto diversa da quella solita delle funzioni d'onda !!!
- 2) riguarda il fare degli esperimenti che danno un risultato definito



**Fig. 3.1** The quantum particle-in-a-box (whose spatial degree of freedom is called  $x$ ) is shown on the *left*; the curve is meant to indicate its wave function (though one should be careful not to take this picture too literally!). Then there is an energy-measurement device which will perform the measurement. The device has a macroscopic pointer, which we can idealize as a single, very heavy particle with horizontal coordinate  $y$ . Prior to the measurement-interaction, the pointer is sitting in its “ready” position ( $y_0$ ); after the measurement interaction, the pointer will move to a new position which indicates the outcome of the measurement:  $y_1$  will mean that the energy of the particle is  $E_1$ , etc



particle in a box



[diapositive]

CASE STRANE (1) dinamiche, 2) ontologia)

1) Durante il periodo della misura la funzione d'onda collassa invece di evolvere secondo l'equazione di Schrödinger. E' come se l'equazione di Schrödinger omettessero di funzionare nel descrivere la funzione d'onda durante una misura.

2) Non si parla di "funzione d'onda del puntatore", per cui l'apparato è descritto in termini classici: è come se i sistemi quantistici esistessero IN AGGIUNTA ai sistemi classici.

Inoltre non è ben specificato quali processi fisici possano essere considerati MISURE.

Bell p. 63-64 [diapositive]

Bell p. 64 [diapositive]

Se gli oggetti macroscopici sono costituiti da tante particelle quantistiche, il mondo MACROSCOPICO non dovrebbe EMERGERE dalla descrizione quantistica del mondo sottile essere POSTULATO?

1. J.S. Bell, Against ‘Measurement’, reprinted in *Speakable and Unspeakable in Quantum Mechanics*, 2nd edn. (Cambridge University Press, Cambridge, 2004)

What exactly qualifies some physical systems to play the role of ‘measurer’? Was the wave-function of the world waiting to jump for thousands of millions of years until a single-celled living creature appeared? Or did it have to wait a little longer, for some better qualified system ... with a Ph.D.? If the theory is to apply to anything but highly idealised laboratory operations, are we not obliged to admit that more or less ‘measurement-like’ processes are going on more or less all the time, more or less everywhere? Do we not have [quantum] jumping [i.e., collapse] then all the time? [1]

2. P.C.W. Davies, R. Brown (eds.), *The Ghost in the Atom*, interview with J.S. Bell (Cambridge University Press, Cambridge, 1986)

In an interview, Bell was once asked whether he thought the problems with quantum mechanics were philosophical or experimental. His answer is relevant here:

I think there are *professional* problems. That is to say, I'm a professional theoretical physicist and I would like to make a clean theory. And when I look at quantum mechanics I see that it's a dirty theory. The formulations of quantum mechanics that you find in the books involve dividing the world into an observer and an observed, and you are not told where that division comes – on which side of my spectacles it comes, for example – or at which end of my optic nerve. You're not told about this division between the observer and the observed. What you learn in the course of your apprenticeship is that for practical purposes it does not much matter where you put this division; that the ambiguity is at a level of precision far beyond human capability of testing. So you have a theory which is fundamentally ambiguous... [2].

## • Trattazione formale

Proviamo ad esplorare queste ipotesi dal punto di vista della meccanica quantistica.

Sia

$$\Psi_0(x) = c_1 \psi_1(x) + c_2 \psi_2(x) + c_3 \psi_3(x)$$

un esempio di uno stato iniziale di una particella in una certa.

[diapositive de fig. 3.2]

Proviamo a descrivere il puntatore in termini quasi-  
meccanici.

Nello stato iniziale:

$$\phi(y) = N e^{-\frac{(y-y_0)^2}{4\sigma^2}},$$

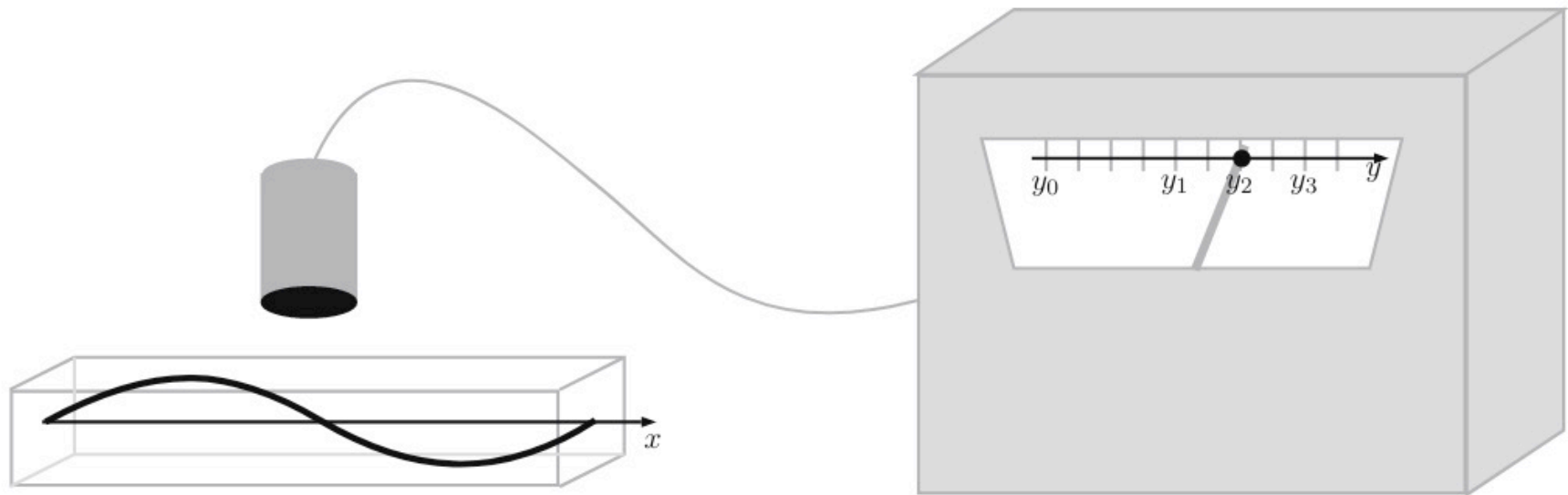
cioè il puntatore è descritto da una funzione gaussiana centrata in  $y_0$ .

Quando le misure cominciano, per esempio a  $t=0$ , potremmo scrivere

$$\Psi_0(x, y) = \psi_0(x) \phi(y)$$

e il sistema quantistico evolverà seguendo:

$$i\hbar \frac{\partial \Psi(x, y, t)}{\partial t} = \hat{H} \Psi(x, y, t)$$



**Fig. 3.2** One of the three possible post-measurement states of the particle-in-a-box and measurement apparatus pointer: the wave function of the PIB has “collapsed” to  $\psi_2$  and the pointer has moved to position  $y_2$ , indicating that the energy measurement had outcome  $E_2$



L'hamiltoniana dovrebbe essere data da tre contributi

$$\hat{H}_x = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x)$$

$$\hat{H}_y = -\frac{\hbar^2}{2M} \frac{\partial^2}{\partial y^2} \approx 0 \quad \text{se } M \text{ è molto grande}$$

$$\hat{H}_{int} = \lambda \hat{H}_x \hat{p}_y = -i\hbar \lambda \hat{H}_x \frac{\partial}{\partial y}$$

\* questo termine dovrebbe far espandere nel tempo il pacchetto d'onde  $\phi(y)$

Questo  $\hat{H}_{int}$  è costruito in modo da,

se la particella si trova in  $\psi_m$  con energia  $E_m$ , il pacchetto si muoverà in una distanza proporzionale a  $E_m$ .

Testiamo il tutto

$$\Psi(x, y, t) = \psi_m(x) \phi(y)$$

Supponiamo, per semplicità, che  $\lambda$  sia molto grande, in modo da poter trascurare, durante l'interazione, gli altri termini dell'hamiltoniana.

$$\Rightarrow i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}_{int} \Psi = -i\hbar \lambda \hat{H}_x \frac{\partial \Psi}{\partial y}$$

$$\Rightarrow \frac{\partial \Psi}{\partial t} = -\lambda \hat{H}_x \frac{\partial \Psi}{\partial y}$$

Supponiamo  $\Psi(x, y, t)$  rimane proporzionale a  $\psi_m(x)$

$$\Rightarrow \frac{\partial \Psi}{\partial t} = -\lambda \hat{H}_x \psi_m(x) \frac{\partial \phi(y)}{\partial y} =$$

$$= -\lambda E_m \frac{\partial}{\partial y} [\psi_m(x) \phi(y)] = -\lambda E_m \frac{\partial \Psi}{\partial y}$$

⇒ SOLUZIONE  $\Psi(x, y, t) = \psi_m(x) \phi(y - \lambda E_m t)$

Se l'interazione (cavità) dura per un tempo  $t = T$  alla fine avremo

$$\Psi(x, y, T) = \psi_m(x) \phi(y - \lambda E_m T)$$

$\uparrow$  stato della particella in cavità.       $\uparrow$  pacchetto gaussiano traslato

⇒ l'hamiltoniana d'interazione fa il lavoro che volevamo fare.

Il puntatore si trova in  $y_m = y_0 + \lambda E_m T$ .

oss. Includere il termine  $\hat{H}_x$  (che abbiamo trascurato) avrebbe avuto, come solo effetto, di moltiplicare per un fattore di fase  $e^{-i E_m t / \hbar}$ , cioè una fase globale.

oss. Includere il termine  $\hat{H}_y$  (anch'esso trascurato) avrebbe avuto il solo effetto di far spezzare un po' il pacchetto d'onda del puntatore.

→ Le due oss. sono due possibili esercizi per casa.

Poiché l'equazione di Schrödinger è lineare, se  $\hat{H} = \hat{H}_{int}$

e  $\Psi(x, y, 0) = \left( \sum_i c_i \psi_i(x) \right) \phi(y)$  avremo, a  $t = T$ ,

$$\Psi(x, y, T) = \sum_i c_i \psi_i(x) \phi(y - \lambda E_i T)$$

SOVRAPPOSIZIONE  
INGARBUGLIATA  
(entanglement)

La particella non ha un'energia definita e il puntatore non è in una sola posizione !!!

## • Il gatto di Schrödinger

Schrödinger pp. 69, 70, 71 [diapositive]

Il gatto. Nel corso dell'ora l'atomo radioattivo evolve verso una sovrapposizione di stati equiprobabili:

$$\Psi_0 \rightarrow \frac{1}{\sqrt{2}} \Psi_{\text{dec}} + \frac{1}{\sqrt{2}} \Psi_{\text{dec}}^{\bar{}}$$

Trattando l'apparato circostante

classicamente, il gatto è vivo <sup>(out)</sup> o morto, e abbiamo bisogno di assumere il collasso delle funzioni d'onda.

Se invece lo trattiamo quantisticamente

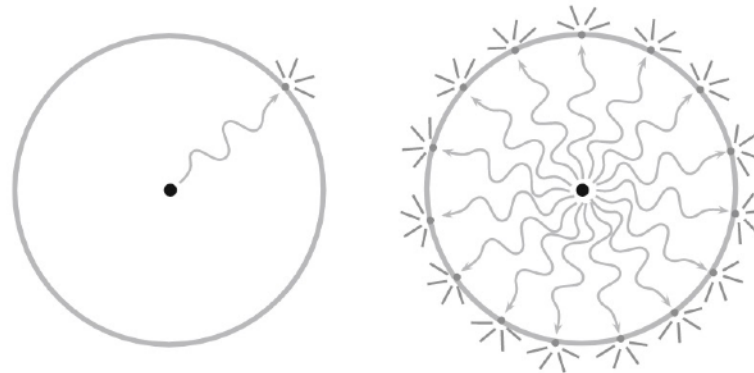
$$\Psi_f = \frac{1}{\sqrt{2}} \Psi_{\text{dec}} \Phi_{\text{rot}} \chi_{\text{morto}} + \frac{1}{\sqrt{2}} \Psi_{\text{dec}}^{\bar{}} \Phi_{\text{rot}}^{\bar{}} \chi_{\text{vivo}}$$

Einstein pp. 72-73 [diapositive]

Il gatto di Bell p. 73 [diapositive]

3. E. Schrödinger, The present situation in quantum mechanics. *Naturwissenschaften* **23** (1935), translated by J. Trimmer, in *Proceedings of the American Philosophical Society*, vol. 124, 10 October 1980 (1980), pp. 323–338

[the  $\psi$ -] function has provided quite intuitive and convenient ideas, for instance the ‘cloud of negative electricity’ around the nucleus, etc. But serious misgivings arise if one notices that the uncertainty affects macroscopically tangible and visible things, for which the term ‘blurring’ seems simply wrong. The state of a radioactive nucleus is presumably blurred in such degree and fashion that neither the instant of decay nor the direction, in which the emitted  $\alpha$ -particle leaves the nucleus, is well-established. Inside the nucleus, blurring doesn’t bother us. The emerging particle is described, if one wants to explain intuitively, as a spherical wave that continuously emanates in all directions from the nucleus and that impinges continuously on a surrounding luminescent screen over its full expanse. **The screen however does not show a more or less constant uniform surface glow, but rather lights up at one instant at one spot – or, to honor the truth, it lights up now here, now there, for it is impossible to do the experiment with only a single radioactive atom.** If in place of the luminescent screen one uses a spatially extended detector, perhaps a gas that is ionised by the  $\alpha$ -particles, one finds the ion pairs arranged along rectilinear columns, that project backwards on to the bit of radioactive matter from which the  $\alpha$ -radiation comes (C.T.R. Wilson’s cloud chamber tracks, made visible by drops of moisture condensed on the ions) [3].



3. E. Schrödinger, The present situation in quantum mechanics. *Naturwissenschaften* **23** (1935), translated by J. Trimmer, in *Proceedings of the American Philosophical Society*, vol. 124, 10 October 1980 (1980), pp. 323–338

One can even set up quite **ridiculous** cases. A cat is penned up in a steel chamber, along with the following diabolical device (which must be secured against direct interference by the cat): in a Geiger counter there is a tiny bit of radioactive substance, *so* small, that *perhaps* in the course of one hour one of the atoms decays, but also, with equal probability, perhaps none; if it happens, the counter tube discharges and through a relay releases a hammer which shatters a small flask of hydrocyanic acid. If one has left this entire system to itself for an hour, one would say that the cat still lives *if* meanwhile no atom has decayed. The first atomic decay would have poisoned it. **The  $\psi$ -function of the entire system would express this by having in it the living and the dead cat (pardon the expression) mixed or smeared out in equal parts [3].**

5. I. Born, trans., *The Born–Einstein Letters* (Walker and Company, New York, 1971)

I am as convinced as ever that this most remarkable situation has come about because we have not yet achieved a complete description of the actual state of affairs.

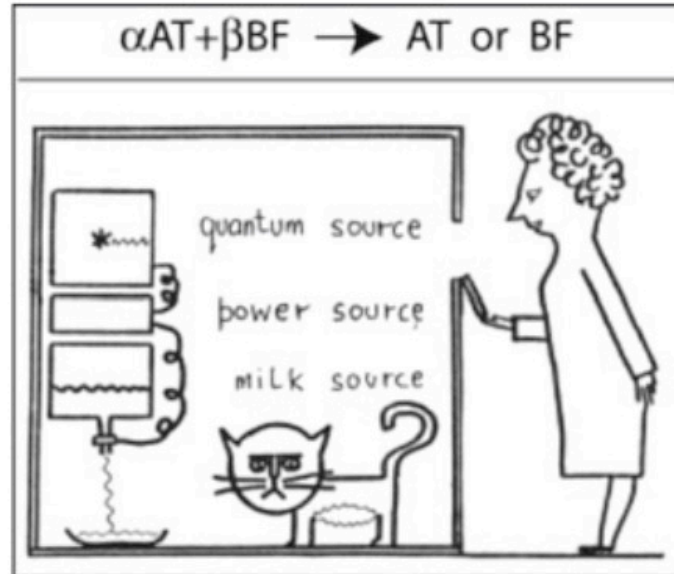
Of course I admit that such a complete description would not be observable in its entirety in the individual case, but from a rational point of view one also could not require this....

Best regards from

Yours, A. Einstein [5, pp. 35-6]

## Il gatto di Bell

6. J.S. Bell, The trieste lecture of John Stewart Bell, transcribed by A. Bassi, G.C. Ghirardi. J. Phys. A: Math. Theor. **40**, 2919–2933 (2007)



**Fig. 3.4** Bell's version of Schrödinger's cat. The state of the radioactive nucleus ("A" for "not decayed" and "B" for "decayed") becomes entangled with the delivery (or not) of milk into the cat's dish and thereby also with the size of the cat's stomach ("T" for "thin" and "F" for "fat"). From Ref. [6]. Figure © IOP Publishing. Reproduced with permission. All rights reserved. <https://doi.org/10.1088/1751-8121/40/12/S02>

- L'interpretazione dell'ignoranza e le variabili nascoste.

seguire il libro

Einstein p. 76 ! [dispositiva]

Nella doppia fenditura  $\Psi = \frac{1}{\sqrt{2}} [\Psi_{f_1} + \Psi_{f_2}]$

⇒ "La funzione d'onda deve essere qualcosa di fisso" (T. Norsen).

- Sintesi su "Il problema delle misure"

1) Non è chiaro quali intervalli e processi fisici continuo come "misure". Questo comporta che non è del tutto chiaro cosa la teoria ci stia dicendo.

2) Anche se la nozione di misura fosse definita, distinguendo per esempio ciò che è misure da ciò che non lo è, dovremmo comunque accettare che il mondo sia diviso in due reami, mentre una teoria fondamentale dovrebbe fornire una visione del mondo unitaria.

[citazioni da Bell e Schrödinger e pp. 79-80]

3) Se rifiutiamo la divisione in 2) la teoria stessa porta a risultati che contraddicono l'esperienza, come nel caso del gatto di Schrödinger.

! p. 80

! p. 80



8. A. Einstein, Reply to criticisms, in *Albert Einstein: Philosopher-Scientist*, ed. by P.A. Schilpp (1949)

Within the framework of statistical quantum theory there is no such thing as a complete description of the individual system. More cautiously it might be put as follows: The attempt to conceive the quantum-theoretical description as the complete description of the individual systems leads to unnatural theoretical interpretations, which become immediately unnecessary if one accepts the interpretation that the description refers to ensembles of systems and not to individual systems. In that case the whole ‘egg-walking’ performed in order to avoid the ‘physically real’ becomes superfluous. There exists, however, a simple psychological reason for the fact that this most nearly obvious interpretation is being shunned. For if the statistical quantum theory does not pretend to describe the individual system (and its development in time) completely, it appears unavoidable to look elsewhere for a complete description of the individual system; in doing so it would be clear from the very beginning that the elements of such a description are not contained within the conceptual scheme of the statistical quantum theory. With this one would admit that, in principle, this scheme could not serve as the basis of theoretical physics. Assuming the success of efforts to accomplish a complete physical description, the statistical quantum theory would, within the framework of future physics, take an approximately analogous position to the statistical mechanics within the framework of classical mechanics. I am rather firmly convinced that the development of theoretical physics will be of this type; but the path will be lengthy and difficult [8, p. 671].

13. J.S. Bell, Against ‘measurement’, in *62 Years of Uncertainty: Erice, 5–14 August 1989* (Plenum Publishers, New York); reprinted in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 2004)

There can be no question then of identifying the quantum system  $S$  with the whole world  $W$ . There can be no question – without changing the axioms – of getting rid of the shifty split. Sometimes some authors of ‘quantum measurement’ theories seem to be trying to do just that. It is like a snake trying to swallow itself by the tail. It can be done – up to a point. But it becomes embarrassing for the spectators even before it becomes uncomfortable for the snake [13].

3. E. Schrödinger, The present situation in quantum mechanics. *Naturwissenschaften* **23** (1935), translated by J. Trimmer, in *Proceedings of the American Philosophical Society*, vol. 124, 10 October 1980 (1980), pp. 323–338

any *measurement* suspends the law that otherwise governs continuous time-dependence of the  $\psi$ -function and brings about in it a quite different change, not governed by any law but rather dictated by the result of the measurement. But laws of nature differing from the usual ones cannot apply during a measurement, for objectively viewed it is a natural process like any other, and it cannot interrupt the orderly course of natural events. Since it does interrupt that of the  $\psi$ -function, the latter ... can *not* serve ... as an experimentally verifiable representation of an objective reality [3].

- Il problema della località

- Le scatole di Einstein

Einstein pp 88-89 [diapositive]

Einstein ci spinge a considerare le conseguenze del concetto  
le funzione d'onda non come un catalogo incompleto di  
informazioni, ma come una descrizione fedele e completa  
di una specie di nuvole diffuse.

Esempio delle scatole

Einstein p. 90 [diapositive]  
" p. 91

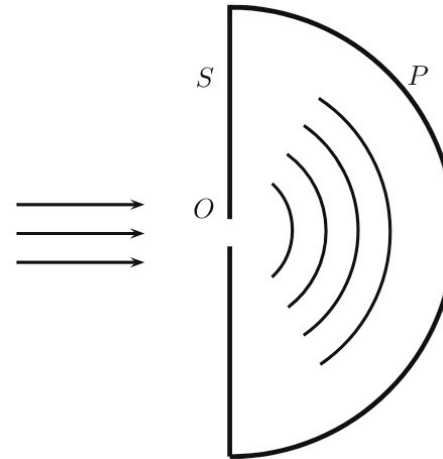
Heisenberg p. 91 [diapositive]

Heisenberg: non c'è trasmissione di segnale super-luminale.  
Qui riprende il libro, p. 92.

De Broglie p. 92-93 [diapositive]

1. Einstein's remarks from Solvay 1927, translated in Bacciogallupi and Valentini, *Quantum Theory at the Crossroads*, pp. 485–487, <http://arxiv.org/pdf/quant-ph/0609184.pdf>

**Fig. 4.1** A single electron approaches a narrow slit ( $O$ ) in a screen ( $S$ ). Downstream of the slit, the wave function diffracts and spreads more or less evenly over a curved detection screen ( $P$ )



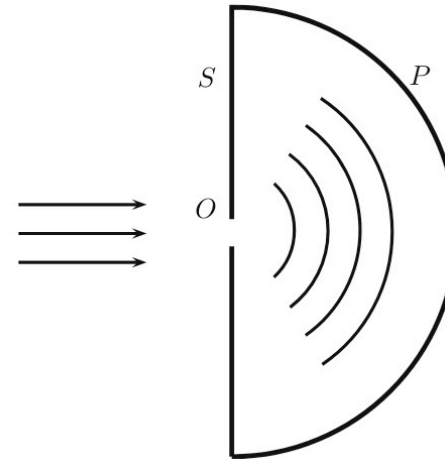
One can take two positions towards the theory with respect to its postulated domain of validity, which I wish to characterise with the aid of a simple example.

Let  $S$  be a screen provided with a small opening  $O$  [see Fig. 4.1] and  $P$  a hemispherical photographic film of large radius. Electrons impinge on  $S$  in the direction of the arrow.... Some of these go through  $O$ , and because of the smallness of  $O$  and the speed of the particles, are dispersed uniformly over the directions of the hemisphere, and act on the film.

Both ways of conceiving the theory now have the following in common. There are de Broglie waves, which impinge approximately normally on  $S$  and are diffracted at  $O$ . Behind  $S$  there are spherical waves, which reach the screen  $P$  and whose intensity at  $P$  is responsible for what happens at  $P$ .

1. Einstein's remarks from Solvay 1927, translated in Bacciogallupi and Valentini, *Quantum Theory at the Crossroads*, pp. 485–487, <http://arxiv.org/pdf/quant-ph/0609184.pdf>

**Fig. 4.1** A single electron approaches a narrow slit ( $O$ ) in a screen ( $S$ ). Downstream of the slit, the wave function diffracts and spreads more or less evenly over a curved detection screen ( $P$ )



We can now characterise the two points of view as follows.

1. Conception I. – The de Broglie - Schrödinger waves do not correspond to a single electron, but to a cloud of electrons extended in space. The theory gives no information about individual processes, but only about the ensemble of an infinity of elementary processes.

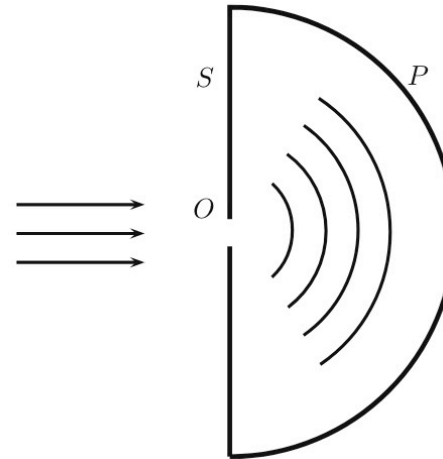
2. Conception II. – The theory claims to be a complete theory of individual processes. Each particle directed towards the screen, as far as can be determined by its position and speed, is described by a packet of de Broglie - Schrödinger waves of short wavelength and small angular width. This wave packet is diffracted and, after diffraction, partly reaches the film  $P$  in a state of resolution.

According to the first, purely statistical, point of view  $|\psi|^2$  expresses the probability that there exists at the point considered *a particular* particle of the cloud, for example at a given point on the screen.

According to the second,  $|\psi|^2$  expresses the probability that at a given instant *the same* particle is present at a given point (for example on the screen). Here, the theory refers to an individual process and claims to describe everything that is governed by laws.

1. Einstein's remarks from Solvay 1927, translated in Bacciogallupi and Valentini, *Quantum Theory at the Crossroads*, pp. 485–487, <http://arxiv.org/pdf/quant-ph/0609184.pdf>

**Fig. 4.1** A single electron approaches a narrow slit ( $O$ ) in a screen ( $S$ ). Downstream of the slit, the wave function diffracts and spreads more or less evenly over a curved detection screen ( $P$ )

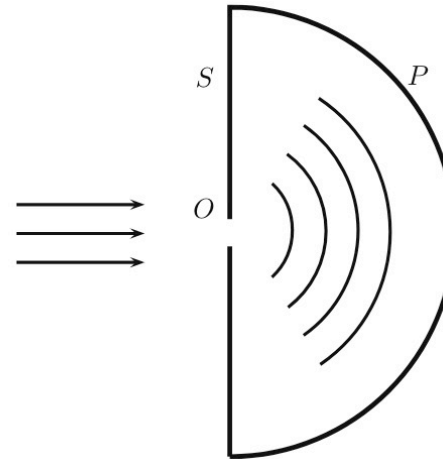


The second conception goes further than the first, in the sense that all the information resulting from I results also from the theory by virtue of II, but the converse is not true. It is only by virtue of II that the theory contains the consequence that the conservation laws are valid for the elementary process; it is only from II that the theory can derive the result of the experiment of Geiger and Bothe, and can explain the fact that in the Wilson [cloud] chamber the droplets stemming from an  $\alpha$ -particle are situated very nearly on continuous lines.

But on the other hand, I have objections to make to conception II. The scattered wave directed towards  $P$  does not show any privileged direction. If  $|\psi|^2$  were simply regarded as

1. Einstein's remarks from Solvay 1927, translated in Bacciogallupi and Valentini, *Quantum Theory at the Crossroads*, pp. 485–487, <http://arxiv.org/pdf/quant-ph/0609184.pdf>

**Fig. 4.1** A single electron approaches a narrow slit ( $O$ ) in a screen ( $S$ ). Downstream of the slit, the wave function diffracts and spreads more or less evenly over a curved detection screen ( $P$ )



the probability that at a certain point a given particle is found at a given time, it could happen that *the same* elementary process produces an action *in two or several* places on the screen. But the interpretation, according to which  $|\psi|^2$  expresses the probability that *this* particle is found at a given point, assumes an entirely peculiar mechanism of action at a distance, which prevents the wave continuously distributed in space from producing an action in *two* places on the screen.

In my opinion, one can remove this objection only in the following way, that one does not describe the process solely by the Schrödinger wave, but that at the same time one localises the particle during the propagation. I think Mr de Broglie is right to search in this direction. If one works solely with the Schrödinger waves, interpretation II of  $|\psi|^2$  implies to my mind a contradiction with the postulate of relativity [1].

2. A. Fine, *The Shaky Game* (University of Chicago Press, Chicago, 1986)

Now I describe a state of affairs as follows: *the probability is 1/2 that the ball is in the first box*. Is this a complete description?

NO: A complete description is: the ball *is* (or is not) in the first box. That is how the characterization of the state of affairs must appear in a complete description.

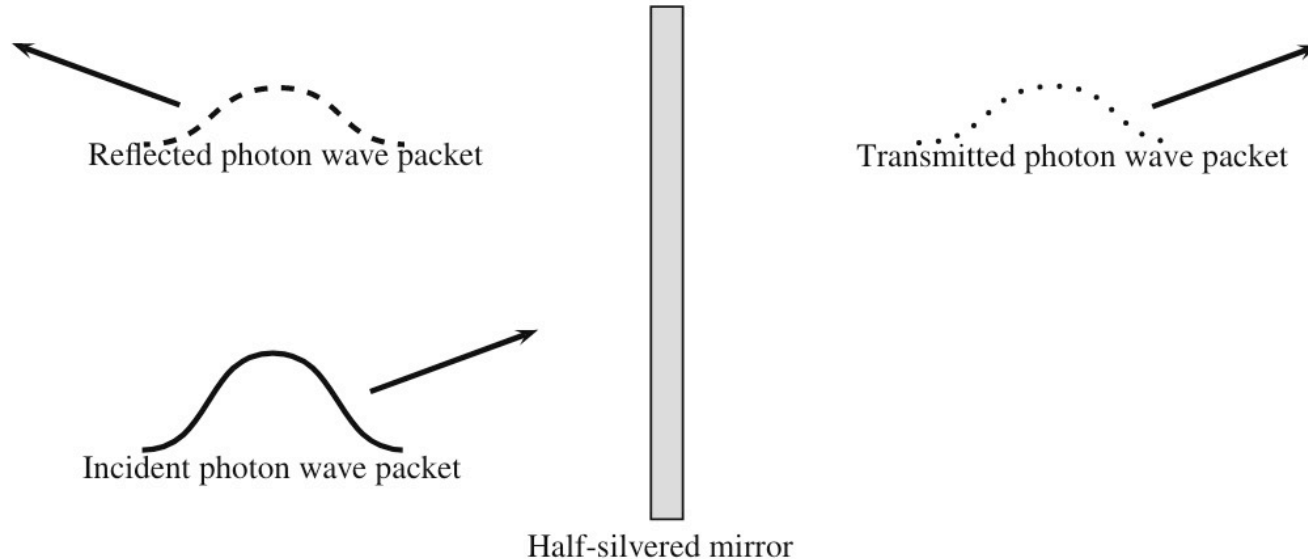
YES: Before I open them, the ball is by no means in *one* of the two boxes. Being in a definite box only comes about when I lift the covers. This is what brings about the statistical character of the world of experience, or its empirical lawfulness. Before lifting the covers the state [of the distant box] is *completely* characterized by the number 1/2, whose significance as statistical findings, to be sure, is only attested to when carrying out observations [2, p. 69].



3. D. Howard, Einstein on locality and separability. *Stud. Hist. Phil. Sci.* **16**, 171–201 (1985)

*My way of thinking is now this: properly considered, one cannot [refute the completeness doctrine, i.e., Conception 2, i.e., the YES view] if one does not make use of a supplementary principle: the ‘separation principle.’ That is to say: ‘the second box, along with everything having to do with its contents, is independent of what happens with regard to the first box (separated partial systems).’ If one adheres to the separation principle, then one thereby excludes the [YES] point of view, and only the [NO] point of view remains, according to which the above state description is an *incomplete* description of reality, or of the real states [3].*

4. W. Heisenberg, *The Physical Principles of the Quantum Theory* (Dover Publications, New York, 1949), p. 39



...one other idealized experiment (due to Einstein) may be considered. We imagine a photon which is represented by a wave packet built up out of Maxwell waves. It will thus have a certain spatial extension and also a certain range of frequency. By reflection at a semi-transparent mirror, it is possible to decompose it into two parts, a reflected and a transmitted packet. There is then a definite probability for finding the photon either in one part or in the other part of the divided wave packet. After a sufficient time the two parts will be separated by any distance desired; now if an experiment yields the result that the photon is, say, in the reflected part of the packet, then the probability of finding the photon in the other part of the packet immediately becomes zero. The experiment at the position of the reflected packet thus exerts a kind of action (reduction of the wave packet) at the distant point occupied by the transmitted packet, and one sees that this action is propagated with a velocity greater than that of light [4, p. 39].

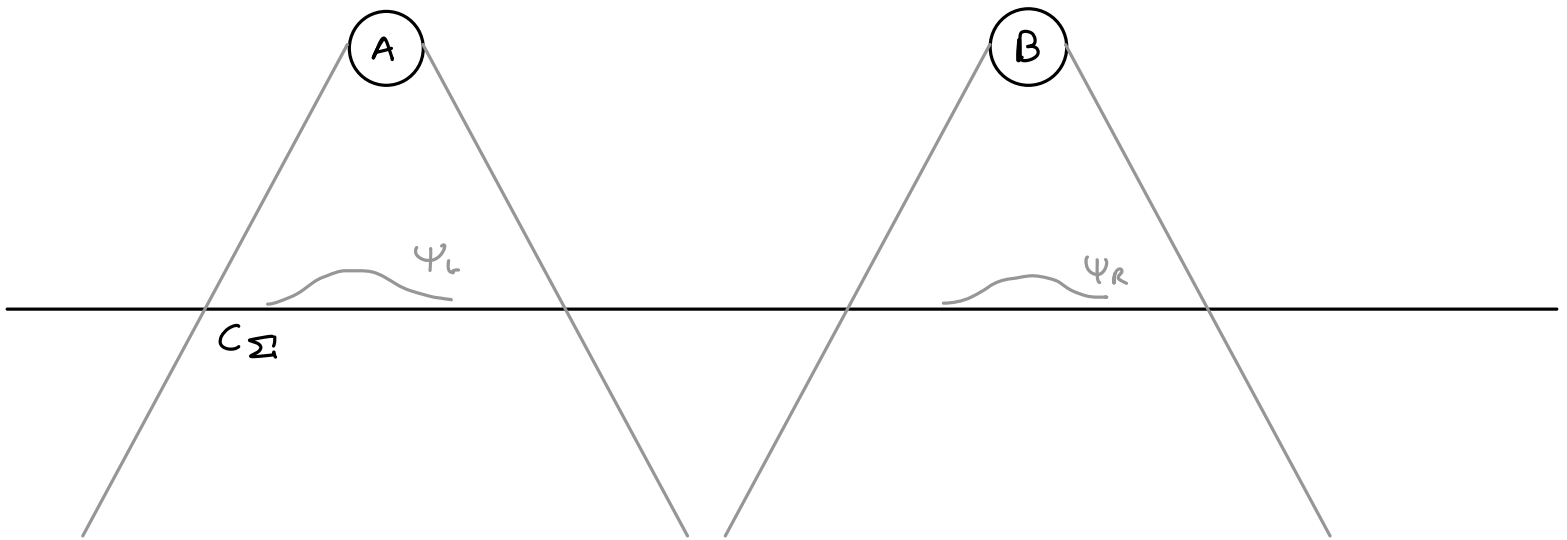
5. L. de Broglie, *The Current Interpretation of Wave Mechanics: A Critical Study* (Elsevier Publishing Company, Amsterdam, 1964)

Suppose a particle is enclosed in a box  $B$  with impermeable walls. The associated wave  $\Psi$  is confined to the box and cannot leave it. The usual interpretation asserts that the particle is ‘potentially’ present in the whole of the box  $B$ , with a probability  $|\Psi|^2$  at each point. Let us suppose that by some process or other, for example, by inserting a partition into the box, the box  $B$  is divided into two separate parts  $B_1$  and  $B_2$  and that  $B_1$  and  $B_2$  are then transported to two very distant places, for example to Paris and Tokyo. The particle, which has not yet appeared, thus remains potentially present in the assembly of the two boxes and its wave function  $\Psi$  consists of two parts, one of which,  $\Psi_1$ , is located in  $B_1$  and the other,  $\Psi_2$ , in  $B_2$ . The wave function is thus of the form  $\Psi = c_1\Psi_1 + c_2\Psi_2$ , where  $|c_1|^2 + |c_2|^2 = 1$ .

5. L. de Broglie, *The Current Interpretation of Wave Mechanics: A Critical Study* (Elsevier Publishing Company, Amsterdam, 1964)

The probability laws of wave mechanics now tell us that if an experiment is carried out in box  $B_1$  in Paris, which will enable the presence of the particle to be revealed in this box, the probability of this experiment giving a positive result is  $|c_1|^2$ , whilst the probability of it giving a negative result is  $|c_2|^2$ . According to the usual interpretation, this would have the following significance: because the particle is present in the assembly of the two boxes prior to the observable localization, it would be immediately localized in box  $B_1$  in the case of a positive result in Paris. This does not seem to me to be acceptable. The only reasonable interpretation appears to me to be that prior to the observable localization in  $B_1$ , we know that the particle was in one of the two boxes  $B_1$  and  $B_2$ , but we do not know in which one, and the probabilities considered in the usual wave mechanics are the consequence of this partial ignorance. If we show that the particle is in box  $B_1$ , it implies simply that it was already there prior to localization. Thus, we now return to the clear classical concept of probability, which springs from our partial ignorance of the true situation. But, if this point of view is accepted, the description of the particle given by the customary wave function  $\Psi$ , though leading to a perfectly *exact* description of probabilities, does not give us a *complete* description of the physical reality, because the particle must have been localized prior to the observation which revealed it, and the wave function  $\Psi$  gives no information about this.

Usiamo ora la località di Bell



Ⓐ si riferisce alle discrepanze del contenuto delle nostre scatole di sinistra

$A = +1$  se le particelle viene trovate a sinistra

$A = 0$  se le nostre scatole di sinistra è vuota

Similmente per B.

$$\Psi = \Psi_L + \Psi_R, \quad \int |\Psi_L|^2 dx = \int |\Psi_R|^2 dx = \frac{1}{2}$$

Secondo la teoria quantistica anche se tutti i dettagli ( $C_\Sigma$ ) fossero specificati, avremmo

$$P[A = +1 | C_\Sigma] = \frac{1}{2}$$

↑

include: pacchetto d'onda  $\Psi_L$ , dettagli fisici del contenuto di sinistra, metodo di trasporto, osservatore, apparato di misura ...

Ma se consideriamo B

$$P[A = +1 | C_{\Sigma}, B = +1] = 0$$

$$P[A = +1 | C_{\Sigma}, B = 0] = 1$$

**viola** la condizione di località di Bell,  
 $P[A | C_{\Sigma}] \neq P[A | C_{\Sigma}, B]$

$\Rightarrow$  Il collasso della funzione d'onda viola la località,  
se consideriamo la funzione d'onda come un campo **REALE**.

Un'assunzione fondamentale in queste analisi è stata che  $\psi_L$  fornisce una descrizione **COMPLETA** del contenuto delle nostre scatole di sinistra.

Le scatole di Einstein ci mostrano quindi che c'è **un dilemma** **tra località e completezza**: se la funzione d'onda fornisce una descrizione completa allora la teoria viola la località. La completezza implica che il collasso della funzione d'onda sia un processo fisso, cosa che però è in conflitto con la teoria della relatività.

Cronaca storica su EPR.

$\hookrightarrow$  Se è incompleta potremmo pensare a variabili nascoste.

Einstein p. 104 [diapositiva]

Bell p. 107-108 [diapositive]

Einstein p. 109-110 [diapositive]

8. A. Einstein, Reply to criticisms, in *Albert Einstein: Philosopher-Scientist*, ed. by P.A. Schilpp (Harper and Row, New York, 1949)

Einstein gave another, much briefer, summary of his position in his other contribution (called “Reply to Criticisms”) to the same 1949 book:

By this way of looking at the matter it becomes evident that the paradox forces us to relinquish one of the following two assertions:

1. the description by means of the  $\psi$ -function is complete.
2. the real states of spatially separated objects are independent of each other.

On the other hand, it is possible to adhere to (2) if one regards the  $\psi$ -function as the description of a (statistical) ensemble of systems (and therefore relinquishes (1)). However, this view blasts the framework of the ‘orthodox quantum theory’ [8].

10. J.S. Bell, Bertlmann's socks and the nature of reality, *Speakable and Unspeakable in Quantum Mechanics*, 2nd edn. (Cambridge University Press, Cambridge, 2004)

The philosopher on the street, who has not suffered a course in quantum mechanics, is quite unimpressed by Einstein–Podolsky–Rosen correlations. He can point to many examples of similar correlations in everyday life. The case of Bertlmann's socks is often cited. Dr. Bertlmann likes to wear two socks of different colours. Which colour he will have on a given foot on a given day is quite unpredictable. But when you see that the first sock is pink you can be already sure that the second sock will not be pink. Observation of the first, and experience of Bertlmann, gives immediate information about the second. There is no accounting for tastes, but apart from that there is no mystery here. And is not the EPR business just the same? [10]



10. J.S. Bell, Bertlmann's socks and the nature of reality, *Speakable and Unsayable in Quantum Mechanics*, 2nd edn. (Cambridge University Press, Cambridge, 2004)

Phenomena of this kind made physicists despair of finding any consistent space-time picture of what goes on on the atomic and subatomic scale. Making a virtue of necessity, and influenced by positivistic and instrumentalist philosophies, many came to hold not only that it is difficult to find a coherent picture but that it is wrong to look for one – if not actually immoral then certainly unprofessional. Going further still, some asserted that atomic and subatomic particles do not *have* any definite properties in advance of observation. There is nothing, that is to say, in the particles approaching the [Stern–Gerlach] magnet, to distinguish those subsequently deflected up from those subsequently deflected down. Indeed even the particles are not really there. [Note: to help prevent the reader from getting lost in quotes within quotes, passages that Bell quotes from other authors are *italicized* in the remainder of this block quote as well as the following one. In particular, the following italicized passages are quotations from Peterson, Heisenberg, Zilsel, Pauli, and Born.]

For example, [Bohr's colleague Peterson recalled that] *Bohr once declared when asked whether the quantum mechanical algorithm could be considered as somehow mirroring an underlying quantum reality: 'There is no quantum world. There is only an abstract quantum mechanical description. It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature'*.

10. J.S. Bell, Bertlmann's socks and the nature of reality, *Speakable and Unsayable in Quantum Mechanics*, 2nd edn. (Cambridge University Press, Cambridge, 2004)

*And for Heisenberg ...in the experiments about atomic events we have to do with things and facts, with phenomena that are just as real as any phenomena of daily life. But the atoms or the elementary particles are not as real; they form a world of potentialities or possibilities rather than one of things or facts.*

*And [Zilsel recollects] Jordan declared, with emphasis, that observations not only **disturb** what has to be measured, they **produce** it. In a measurement of position, for example, as performed with the gamma ray microscope, 'the electron is forced to a decision. We compel it **to assume a definite position**; previously it was, in general, neither here nor there; it had not yet made its decision for a definite position... If by another experiment the **velocity** of the electron is being measured, this means: the electron is compelled to decide itself for some exactly defined value of the velocity... we ourselves produce the results of measurement'.*

It is in the context of ideas like these that one must envisage the discussion of the Einstein–Podolsky–Rosen correlations. Then it is a little less unintelligible that the EPR paper caused such a fuss, and that the dust has not settled even now. It is as if we had come to deny the reality of Bertlmann's socks, or at least of their colours, when not looked at. And as if a child had asked: How come they always choose different colours when they *are* looked at? How does the second sock know what the first has done?

10. J.S. Bell, Bertlmann's socks and the nature of reality, *Speakable and Unsayable in Quantum Mechanics*, 2nd edn. (Cambridge University Press, Cambridge, 2004)

Paradox indeed! But for the others, not for EPR. EPR did not use the word 'paradox'. They were with the man in the street in this business. For them these correlations simply showed that the quantum theorists had been hasty in dismissing the reality of the microscopic world. In particular Jordan had been wrong in supposing that nothing was real or fixed in that world before observation. For after observing only one particle the result of subsequently observing the other (possibly at a very remote place) is immediately predictable. Could it be that the first observation somehow fixes what was unfixed, or makes real what was unreal, not only for the near particle but also for the remote one? For EPR that would be an unthinkable 'spooky action at a distance'. To avoid such action at a distance they have to attribute, to the space-time regions in question, *real* properties in advance of observation, correlated properties, which *predetermine* the outcomes of these particular observations. Since these real properties, fixed in advance of observation, are not contained in quantum formalism, that formalism for EPR is *incomplete*. It may be correct, as far as it goes, but the usual quantum formalism cannot be the whole story [10].

10. J.S. Bell, Bertlmann's socks and the nature of reality, *Speakable and Unsayable in Quantum Mechanics*, 2nd edn. (Cambridge University Press, Cambridge, 2004)

It is important to note that to the limited degree to which *determinism* plays a role in the EPR argument, it is not assumed but *inferred*. What is held sacred is the principle of 'local causality' – or 'no action at a distance'. Of course, mere *correlation* between distant events does not by itself imply action at a distance, but only correlation between the signals reaching the two places. These signals, in the idealized example of Bohm, must be sufficient to *determine* whether the particles go up or down. For any residual indeterminism could only spoil the perfect correlation.

It is remarkably difficult to get this point across, that determinism is not a *presupposition* of the analysis. There is a widespread and erroneous conviction that for Einstein determinism was always *the* sacred principle. The quotability of his famous 'God does not play dice' has not helped in this respect. Among those who had great difficulty in seeing Einstein's position was Born. Pauli tried to help him in a letter of 1954:

*...I was unable to recognize Einstein whenever you talked about him in either your letter or your manuscript. It seemed to me as if you had erected some dummy Einstein for yourself, which you then knocked down with great pomp. In particular, Einstein does not consider the concept of 'determinism' to be as fundamental as it is frequently held to be (as he told me emphatically many times)... he **disputes** that he uses as a criterion for the admissibility of a theory the question: 'Is it rigorously deterministic?' ... he was not at all annoyed with you, but only said you were a person who will not listen.*

10. J.S. Bell, Bertlmann's socks and the nature of reality, *Speakable and Unsayable in Quantum Mechanics*, 2nd edn. (Cambridge University Press, Cambridge, 2004)

Born had particular difficulty with the Einstein–Podolsky–Rosen argument. Here is his summing up, long afterwards, when he edited the Born-Einstein correspondence:

*The root of the difference between Einstein and me was the axiom that events which happen in different places A and B are independent of one another, in the sense that an observation on the state of affairs at B cannot teach us anything about the state of affairs at A.*

Misunderstanding could hardly be more complete. Einstein had no difficulty accepting that affairs in different places could be correlated. What he could not accept was that an intervention at one place could *influence*, immediately, affairs at the other.

These references to Born are not meant to diminish one of the towering figures of modern physics. They are meant to illustrate the difficulty of putting aside preconceptions and listening to what is actually being said. They are meant to encourage *you*, dear listener, to listen a little harder [10].

## 11. A. Einstein, Quantum Mechanics and Reality. Dialectica (1948)

If one asks what, irrespective of quantum mechanics, is characteristic of the world of ideas in physics, one is first of all struck by the following: the concepts of physics relate to a real outside world.... It is further characteristic of these physical objects that they are thought of as arranged in a space-time continuum. An essential aspect of this arrangement of things in physics is that they lay claim, at a certain time, to an existence independent of one another, provided these objects 'are situated in different parts of space.'

The following idea characterizes the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B...

## 11. A. Einstein, Quantum Mechanics and Reality. Dialectica (1948)

There seems to me no doubt that those physicists who regard the descriptive methods of quantum mechanics as definitive in principle would react to this line of thought in the following way: they would drop the requirement ... for the independent existence of the physical reality present in different parts of space; they would be justified in pointing out that the quantum theory nowhere makes explicit use of this requirement.

I admit this, but would point out: when I consider the physical phenomena known to me, and especially those which are being so successfully encompassed by quantum mechanics, I still cannot find any fact anywhere which would make it appear likely that (that) requirement will have to be abandoned.

I am therefore inclined to believe that the description of quantum mechanics ... has to be regarded as an incomplete and indirect description of reality, to be replaced at some later date by a more complete and direct one [11].

## • Il problema dell'ontologia

### • Complessità e Realtà fisica

$\Psi$  è complesso e questo non è sorprendente poiché è  
appare nell'equazione di Schrödinger

[diapositiva]

Resnick p. 115  $\rightarrow$  se questo fosse vero dovremmo rifiutare  
l'idea che la funzione d'onda ci dia una descrizione completa  
della realtà fisica. Tuttavia,

non è vero che siamo obbligati a usare numeri complessi,  
infatti:

$$\Psi(x, t) = f(x, t) + i g(x, t)$$

con  $f, g$  funzioni reali

$$\Rightarrow \begin{cases} -\hbar \frac{\partial g}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 f + V f \\ \hbar \frac{\partial f}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 g + V g \end{cases}$$

Questo potrebbe essere un modo per considerare le funzioni  
d'onda come qualcosa di reale: esse rappresentate due  
campi accoppiati.

Per esempio, nell'elettromagnetismo nello spazio vuoto,

$$\vec{\nabla} \cdot \vec{E} = 0, \quad \vec{\nabla} \cdot \vec{B} = 0, \quad \vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad \vec{\nabla} \wedge \vec{B} = \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t}$$

scirendo  $\vec{F} = \vec{E} + i c \vec{B}$

$$\Rightarrow \vec{\nabla} \cdot \vec{F} = 0$$

$$\vec{\nabla} \wedge \vec{F} = \frac{i}{c} \frac{\partial \vec{F}}{\partial t}$$

che si può scrivere come  $i \hbar \frac{\partial \vec{F}}{\partial t} = \hbar c \vec{\nabla} \wedge \vec{F}$



1. R. Eisberg, R. Resnick, *Quantum Physics*, 2nd edn. (Wiley, New York, 1985)

The fact that wave functions are complex functions should not be considered a weak point in the quantum mechanical theory. Actually, it is a desirable feature because it makes it immediately apparent that we should not attempt to give to wave functions a physical existence in the same sense that water waves have a physical existence. The reason is that a complex quantity cannot be measured by any actual physical instrument. The ‘real’ world (using the term in its nonmathematical sense) is the world of ‘real’ quantities (using the term in its mathematical sense) [1, p. 134].

cioè di tipo Schrödinger

$$i\hbar \frac{\partial \vec{F}}{\partial t} = \hat{H} \vec{F} \quad \text{con} \quad \hat{H} = \hbar c \vec{\nabla} \wedge$$

questo non vuol dire che  $\vec{F}$  non sia per niente fisicamente reale, poiché è composta di  $\vec{E}$  e  $\vec{B}$  che sono fisicamente reali.

Quindi è anche possibile che  $\Psi$ , nonostante sia complessa, rappresenti qualche campo fisicamente reale.

### • Spesso delle configurazioni

Tuttavia la difficoltà sulla realtà fisica delle funzioni d'onda resta poiché si tratta di una funzione nello spazio delle configurazioni.

Lorentz p. 119, note di Morcen [diapositiva]

Einstein p. 119 [diapositiva]

Schrödinger pp. 119-120 [diapositiva]

Può essere difficile affermare il problema, poiché si tratta comunque di un'equazione d'onda.

Tuttavia, mentre posso sempre calcolare  $\vec{E}(x,t)$ ,  $\vec{B}(x,t)$  nello spazio e nel tempo, nella maggior parte dei casi (più di una particella) la  $\Psi$  è nello spazio delle configurazioni: non  $\Psi(\vec{x},t)$  ma  $\Psi(\vec{x}_1, \vec{x}_2, \dots, \vec{x}_N, t)$ .

Quindi se la funzione d'onda descrive qualcosa di fisicamente reale, allora la descrizione che fornisce è in qualche modo indiretta o estratta.

! p. 122

2. K. Przibram, *Letters on Wave Mechanics*, Martin Klein, trans (Philosophical Library, NY, 1967)

If I had to choose now between your wave mechanics and the matrix mechanics, I would give the preference to the former, because of its greater intuitive clarity, so long as one only has to deal with the three coordinates  $x$ ,  $y$ ,  $z$ . If, however, there are more degrees of freedom, then I cannot interpret the waves and vibrations physically, and I must therefore decide in favor of matrix mechanics. But your way of thinking has the advantage for this case too that it brings us closer to the real solution of the equations; the eigenvalue problem is the same in principle for a higher dimensional  $q$ -space as it is for a three dimensional space [2, p. 43–44].

Lorentz

3. D. Howard, Nicht Sein Kann Was Nicht Sein Darf, or the Prehistory of EPR, 1909–1935: Einstein’s Early Worries about the Quantum Mechanics of Composite Systems, in *Sixty-Two Years of Uncertainty*, ed. by A.I. Miller (Plenum Press, New York, 1990)

Einstein expressed a similar concern about Schrödinger’s wave function in letters from this same period. Here are some excerpts, all quoted in Ref. [3]:

- “Schrödinger’s conception of the quantum rules makes a great impression on me; it seems to me to be a bit of reality, however unclear the sense of waves in n-dimensional q-space remains”. (May 1, 1926, to Lorentz)
- “Schrödinger’s works are wonderful – but even so one nevertheless hardly comes closer to a real understanding. The field in a many-dimensional coordinate space does not smell like something real.” (June 18, 1926, to Ehrenfest)
- “The method of Schrödinger seems indeed more correctly conceived than that of Heisenberg, and yet it is hard to place a function in coordinate space and view it as an equivalent for a motion. But if one could succeed in doing something similar in four-dimensional space, then it would be more satisfying.” (June 22, 1926, to Lorentz)
- “Of the new attempts to obtain a deeper formulation of the quantum laws, that by Schrödinger pleases me most. If only the undulatory fields introduced there could be transplanted from the n-dimensional coordinate space to the 3 or 4 dimensional!” (August 21, 1926, to Sommerfeld)
- “Schrödinger is, in the beginning, very captivating. But the waves in n-dimensional coordinate space are indigestible...” (August 28, 1926, to Ehrenfest)
- “The quantum theory has been completely Schrödingerized and has much practical success from that. But this can nevertheless not be the description of a real process. It is a mystery.” (February 16, 1927, to Lorentz)

4. G. Bacciogallupi, A. Valentini, *Quantum Theory at the Crossroads*, <http://arxiv.org/pdf/quant-ph/0609184.pdf>

What does the  $\psi$ -function mean now, that is, *how does the system described by it really look like in three dimensions?* Many physicists today are of the opinion that it does not describe the occurrences in an individual system, but only the processes in an ensemble of very many like constituted systems that do not sensibly influence one another and are all under the very same conditions. I shall skip this point of view since others are presenting it. I myself have so far found useful the following perhaps somewhat naive but quite concrete idea. The classical system of material points does not really exist, instead there exists something that continuously fills the entire space and of which one would obtain a ‘snapshot’ if one dragged the classical system, with the camera shutter open, through *all* its configurations, the representative point in  $q$ -space spending in each volume element  $d\tau$  a time that is proportional to the *instantaneous* value of  $\psi\psi^*$ . (The value of  $\psi\psi^*$  for only *one* value of the argument  $t$  is thus in question.) Otherwise stated: the real system is a superposition of the classical one in all its possible states, using  $\psi\psi^*$  as ‘weight function’ [4, p. 453].

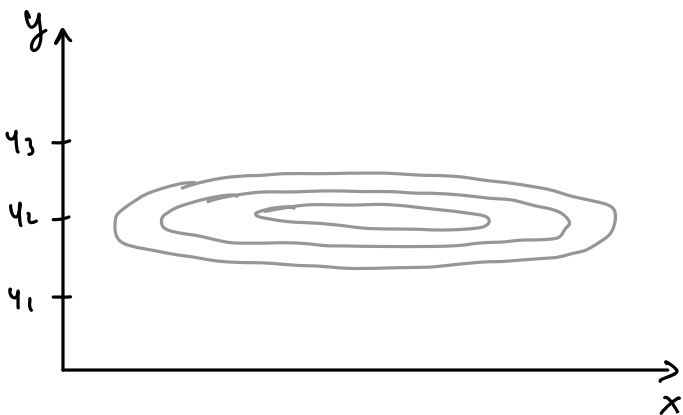
## • Ontologie, Misura e Località

Riprendendo l'esempio delle misure

$$\Psi(x, y, T) = \psi_m(x) \phi(y - \lambda E_m T)$$

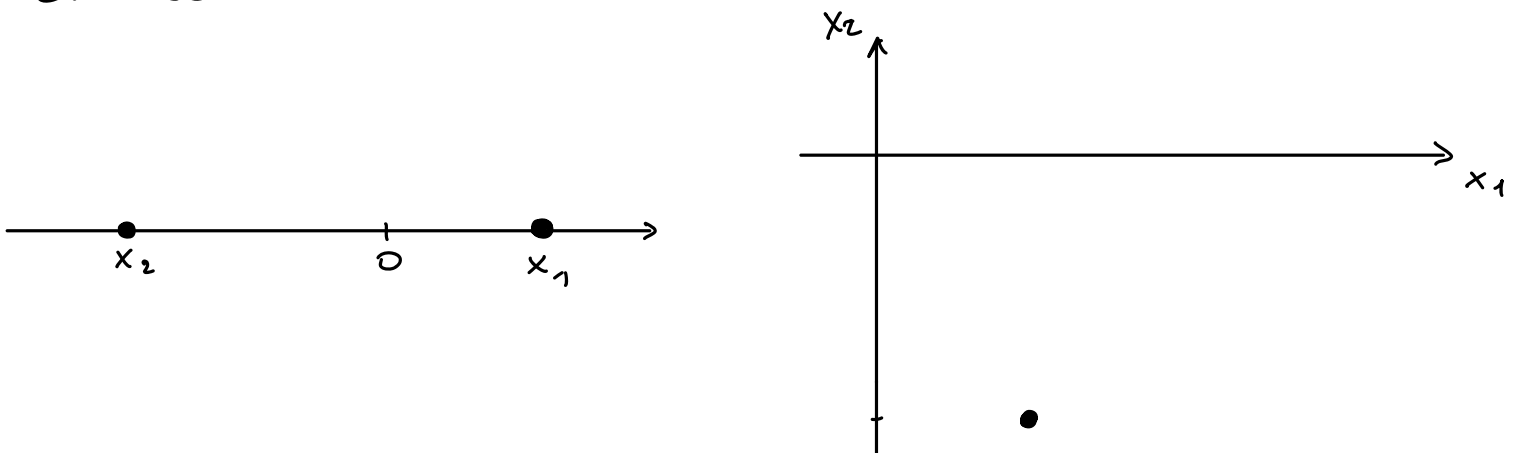
- 1) non è detto che la funzione d'onda totale si possa scrivere sempre come un prodotto
- 2) se prendiamo la descrizione quantomeccanica alle lettere, allora lo stato del sistema particella-puntatore è dato semplicemente da  $\Psi$

Nello spazio delle configurazioni:



non sembra una particella in  
una cavità e/o un puntatore

Si potrebbe insistere che la funzione d'onda è una descrizione indiretta e astratta di quello che succede. D'altronde è lo stesso nello spazio delle configurazioni in meccanica classica:



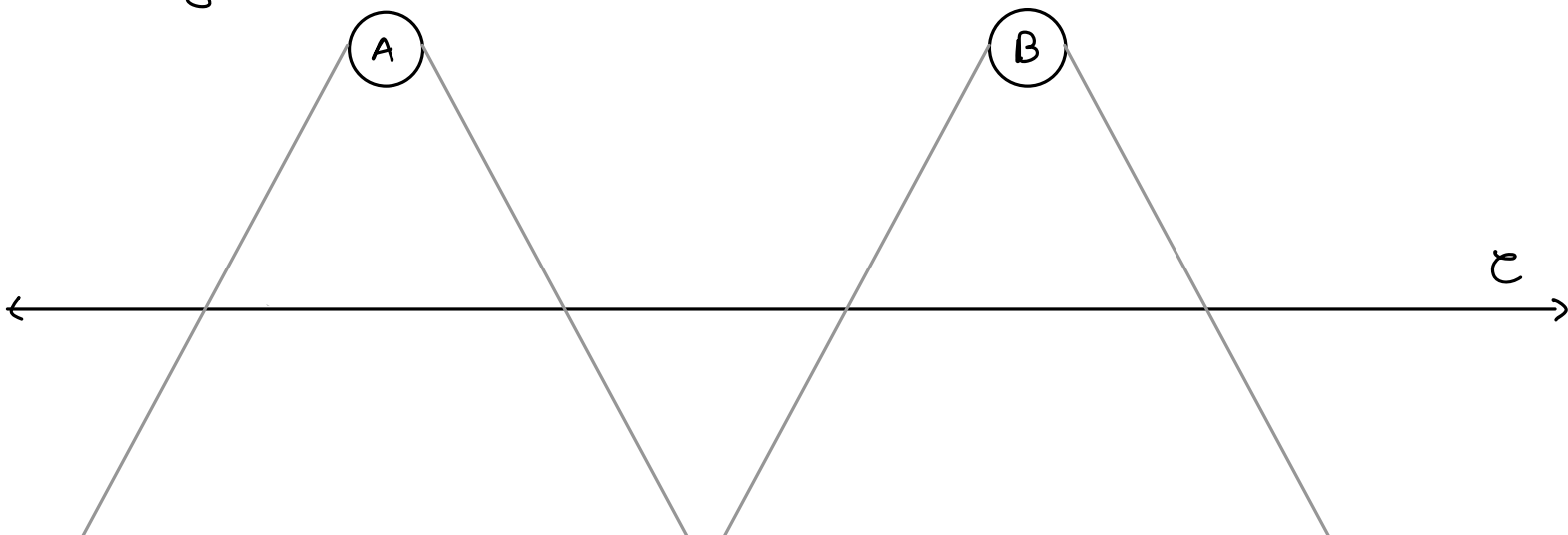
La differenza è che in meccanica classica quella dello spazio delle configurazioni è una descrizione alternativa a quella nello spazio reale, in cui l'ontologia delle particelle è chiara, mentre in meccanica quantistica abbiamo solo la descrizione astratta dello spazio delle configurazioni.

! p. 124

→

Ritornando alla località

- 1) Con le scatole di Einstein abbiamo stabilito che la meccanica quantistica, se assunta come teoria completa, è non-locale.
- 2) Probabilmente la meccanica quantistica non può nemmeno essere definita non-locale, perché non fornisce un'ontologia chiara di oggetti che si muovono e interagiscono. La non-località della meccanica newtoniana è più chiara.
- 3) Seguendo le formulazioni di Bell della località



$$P[A|C] = P[A|C, B]$$

supponiamo che  $C$  sia una specificazione completa di tutti gli eventi dell'universo in un istante di tempo prima di  $A$  e  $B$ .

Se l'equazione di sopra è violata, per qualche teoria, anche specificando  $C$ , allora certamente la teoria è non-locale.

Per esempio  $C = \Psi(x_1, x_2, t)$

$$P[A|\Psi] = P[A|\Psi, B] \quad \text{ma questo non è vero sperimentalmente,}$$

quindi la teoria è non-locale.

$\Rightarrow$  C'è un problema con le funzioni d'onda di molte particelle se sono considerate come descrizioni complete dei sistemi a cui si riferiscono.

Quindi le funzioni d'onda cosa descrivono?

Qualcuno talvolta suggerisce che la teoria quantistica dei campi risolve il problema fornendo una ontologia senza contraddizioni. Tuttavia il problema persiste: è semplicemente spostato sui campi.

p. 134 [diapositiva]



A second and deeper reason for the confusion, though, is just that most people have not really thought carefully about these kinds of issues, even in the context of NRQM. Perhaps they tend to think exclusively about one-particle examples, and so have in mind an ontology of single-particle waves running around through physical space. Or perhaps they do hold some naive version of the “ignorance interpretation”, according to which the ontology is something like classical (i.e., literal) particles, with wave functions providing only some kind of very incomplete description of their states. Or perhaps they don’t have any particular ontological picture in mind, but are instead happy to just play games with mathematical symbols without thinking about (and without even acknowledging that *someone* should think about) what the symbols correspond to in physical reality. In any case, and whatever the ultimate reasons, most physicists have simply not appreciated or accepted that there is some problem associated with understanding what NRQM wave functions might describe exactly – and so they are open to the (in fact rather ridiculous) suggestion that there is definitely no such problem in quantum field theory.

In sintesi:

Fisici "realisti" come Einstein e Schrödinger sembravano aver concluso che le funzioni d'onda non potevano essere una descrizione completa della realtà.

1) Il problema delle **MISURA** mostra che le funzioni d'onda, soluzioni dell'equazione di Schrödinger per ogni  $t$ , sembrano incapaci di catturare i risultati definiti che si osservano quando si misura qualcosa.

2) Einstein et al. hanno mostrato che assumere la completezza delle teorie quantistica con l'idea di causalità **LOCALE** relativistica.

3) Supponendo di trascurare (o risolvere) 1) e 2) resta non chiara l'**ONTOLOGIA** delle funzioni d'onda, cioè se esse descrivono direttamente e fedelmente una realtà fisica tridimensionale, poiché in generale esse sono funzioni su uno spazio astratto (spazio delle configurazioni multidimensionale).